Geometric Modeling

Assignment sheet #9

"(Rational) Spline Curves"

(due July 3rd 2012 before the lecture)

Silke Jansen, Ruxandra Lasowski,
Art Tevs, Michael Wand

Exercise 1 (Spline Curves): **[1+2+1points]**

Consider a **quadratic** B-Spline curve with knot sequence 0, 1/5, 2/5, 3/5, 4/5, 1 and control points:

$$P_0 = P_4 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \quad P_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad P_3 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

a. How many segments does the curve have? What is the interval of the curve ? Why?
   What are the polar forms of the individual segments?
   Sketch the control polygon and the segments

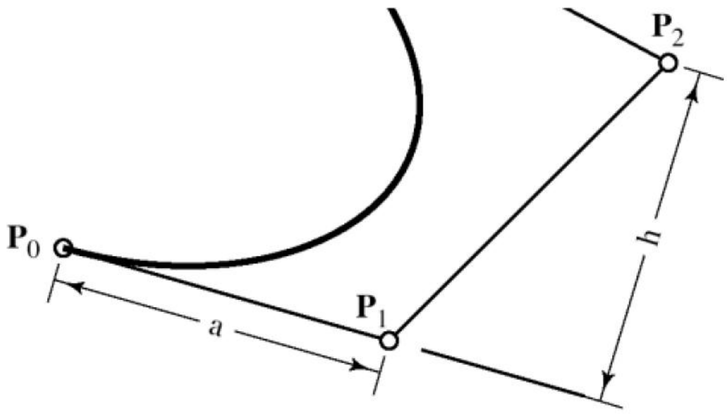
b. Evaluate the curve at t=0.5 using above knot sequence.
   Evaluate the curve at t=0.5 using knot sequence 0, 0, 0.2, 0.5, 1, 1 .

c. Compute the control points for the derivate of the curve and sketch the resulting control polygon.

Exercise 2 (Curavture of Bézier Curves) **[4 points]**

Prove that the curvature of a Bézier curve of degree n at the starting point P_0 is given by:

$$\kappa 2(P_0) = 2\frac{n-1}{n}\frac{area(P_0,P_1,P_2)}{dist^3(P_0,P_1)} = \frac{n-1}{n}\frac{h}{a^2}$$



Exercise 3 (Bézier Curves): [4 points]

Find a cubic Bézier curve P(u), $P:[0,1] \rightarrow R^2$ with:

$$P(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad P(1) = \begin{pmatrix} 9 \\ 0 \end{pmatrix}$$

which intersects itself at $P(\frac{1}{4}) = P(\frac{3}{4})$ orthogonally.

Exercise 4 (Spline Representation of Circles): [3 points]

Prove that a circle $\{(x, y) \in \Re^2 : x^2 + y^2 = r^2\}$ cannot be represented as a non-rational polynomial B-Spline curve. Why is your proof not applicable for rational splines?

Hint: The proof can be done by induction.

Exercise 4 (Reparametrization of Rational Bézier): [1+2+1 points]

a. Let $\gamma > 0$. Prove that $\phi(t) := \dfrac{\gamma t}{1-(1-\gamma)t}$ is a parameter transformation of the interval [0,1], i.e. $\phi(0) = 0$, $\phi(1) = 1$, and $\phi : [0,1] \rightarrow [0,1]$ bijective.

b. Given is a rational Bézier curve *F* with control points $b_0, b_1, \ldots, b_n$ and weights $w_0, w_1, \ldots, w_n$. In addition, let $\tilde{F}(t) := F(\phi(t))$ with $\phi(t)$ as defined in (a).

   Prove that $\tilde{F}$ is a rational Bézier curve as well and find its control points $\tilde{b}_i$ and weights $\tilde{w}_i$.

   Hint: Consider the parameter transformation ϕ for a Bernstein polynomial B_i^n first.

c. Prove that any rational Bézier curve can be normalized by reparametrization such that $w_0 = w_n = 1$.